Transformation of phase transitions driven by an anisotropic random field

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We carry out a comparative study of the influence of a random anisotropy field on continuous and discontinuous phase transitions. The ordered phase, which is reached via a continuous symmetry breaking phase transition, is characterized by an order parameter and by a corresponding hydrodynamic continuum field. We assume that the response of the hydrodynamic field to the imposed disorder results in a domainlike pattern of the system. For a strong enough disorder both transitions become gradual. For weaker disorder strengths the disorder converts a second order transition into a discontinuous one.

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For years there has been considerable interest in the influence of randomness on the phase behavior of condensed matter systems. The pioneering studies were mostly modeling magnetic systems. Imry and Wortis [1] have shown that quenched disorder can smooth a first order phase transition realized in a pure system. For a second order phase transition the Harris criterion [2] is widely known. It claims that quenched disorder is a relevant perturbation at a second order phase transition if $\alpha > 0$ in the pure system. Here α stands for the critical exponent for the specific heat. Imry and Ma [3] have shown that phases reached via a continuous symmetry breaking phase transition are particularly susceptible to disorder.

In recent years various liquid crystal (LC) phases either confined to different porous materials or mixed with aerosil particles were chosen [4] as paradigm systems to study the influence of disorder. The reason is their experimental accessibility arising due to their liquid, soft, and anisotropic optic properties. The randomness in these systems is introduced mainly by randomly varying the orientational anchoring conditions at a LC-pore or LC-aerosil interface. Such binary systems, where the phase behavior of the LC component is perturbed by either inert porous matrices or aerosil particles, are roughly characterized by a characteristic geometrically imposed length l_0. Typically l_0 ranges between nanometer and micrometer sizes. In the case of porous matrices l_0 stands for an average linear void size [5,6] of the system and for aerosils [7] it reveals the average separation between particles. It is widely believed that with decreasing l_0 the degree of randomness increases [6–8]. Most studies so far have focused on the first order isotropic-nematic (I-N) and the second order nematic–smectic-A (N–Sm-A) phase transition. In these cases the continuous symmetry in the orientational (I-N) and translational (N–Sm-A) degree of ordering is broken. Experiments [4–8] show that below critical values $l_0 \equiv l_c$ phase transitions become gradual, where l_c is comparable to the relevant order parameter length. Further with decreasing l_0, where $l_0 > l_c$, the phase transition temperatures become increasingly depressed and phase transition singularities broadened. However the interpretation of experimental results is rather difficult, because they reflect the interplay among randomness, interface wetting, and finite size effects [9]. From the theoretical and numerical side the influence of randomness on such systems was studied using random field [10] and random anisotropy [10–13] methods, where several problems remain open. Further theoretical and numerical studies are needed for a more detailed understanding of realistic systems.

In this paper we carry out a comparative study on the influence of noncorrelated random anisotropy type disorder on the (i) first and (ii) second order continuous symmetry breaking phase transitions of an elastic system using a simple mean-field approach. For perturbed LC phases these cases mimic well the (i) I-N and (ii) N–Sm-A phase transitions. We show that in both cases a strong enough disorder converts the transition into a gradual evolution of the order as a transition driving parameter is varied. Below this threshold, for the case of the continuous phase transition in a pure system, the disorder converts the transition into a discontinuous one. The plan of the paper is as follows. We first introduce the free energy of the system and representative continuum fields. We write down an effective expression for the free energy, where the influence of disorder is presented via the domain structure of the system. Finally we calculate the phase diagram of the system and discuss the results.

We consider an elastic system perturbed by a noncorrelated random anisotropy type of disorder. The phase ordering and structure of the ordered phase are described with a nonconserved order parameter ϕ and a continuum field revealing a long range order of the ordered phase, which we describe by a unit vector field $\vec{m}$. The phase transition is driven by a scalar field t (e.g., the temperature). We assume that (i) at the phase transition the continuous symmetry of the parent phase, i.e., of the field $\vec{m}$, is broken. Further (ii), the newly reached bulk phase is in equilibrium characterized by a homogeneous equilibrium ordering in ϕ and $\vec{m}$. Consequently the field $\vec{m}$ exhibits a Goldstone mode and is classified as the hydrodynamic field (i.e., the energy costs of long wavelength λ field excitations vanish in the limit $\lambda \to \infty$). We further

introduce a random anisotropy field which randomly varies from a site to site.

In the case of perturbed LC phases this description can mimic the isotropic-nematic or the nematic–smectic-*A* phase transition. The simplest representative continuum fields, describing the ordering in the ordered LC phases, are the following. The orientational ordering in the nematic phase is given by the nematic director field $\vec{n}$ and the uniaxial order parameter field S. Here $\vec{n}(\vec{r})$ points along the average orientation of a LC molecule at a point denoted by $\vec{r}$ and $S(\vec{r})$ describes the extent of fluctuations about that direction. The translational ordering in the smectic-*A* phase is conventionally described with the complex order parameter $\psi=\eta e^{i\varphi}$. The translational order parameter $\eta(\vec{r})$ describes the extent of layering and the position of layers is given by the phase factor $\varphi(\vec{r})$. In the case of weakly perturbed layers one conventionally expresses the phase factor with the displacement field $\vec{u}$: $\varphi=\vec{q}_0\cdot(\vec{r}+\vec{u})$. Here the wave vector $\vec{q}_0$ describes the equilibrium layer stacking. Therefore for the (i) *I*-*N* and (ii) *N*–Sm-*A* phase transition the role of the field pair $(\phi,\vec{m})$ is played by (i) $(S,\vec{n})$ and (ii) $(\eta,\vec{u})$, respectively.

The corresponding dimensionless free energy density f of a three-dimensional system is expressed as $f=f_b+f_e+f_{\mathrm{ARF}}$, where the terms f_b, f_e, and f_{ARF} describe the bulk, elastic, and random anisotropy contribution to f.

We express the bulk term as either

$$f_b \equiv f_b^{(1)} = t\phi^2 + \phi^2(1-\phi)^2 \tag{1}$$

or

$$f_b \equiv f_b^{(2)} = \frac{1}{2}t\phi^2 + \frac{1}{4}\phi^4. \tag{2}$$

Here $f_b^{(i)}$ triggers in the bulk and for $f_e=f_{\mathrm{ARF}}=0$ either the first $(i=1)$ or second $(i=2)$ order phase transition at $t\equiv t_c=0$. We name the $t>t_c$ and $t<t_c$ phases as the *isotropic* and *ordered* phases. We also henceforth refer to the $i=1$ and 2 options as the discontinuous and continuous cases, respectively.

The equilibrium phase behavior for $f\equiv f^{(i)}=f_b^{(i)}$ in bulk is the following. For the continuous case the equilibrium order parameter equals $\phi_b(t>t_c)=0$ and $\phi_b(t<t_c)=\sqrt{-t}$. For the discontinuous we have $\phi_b(t>t_c)=0$ and $\phi_b(t<t_c)=\frac{3}{4}(1+\sqrt{1-8(1+t)/9})$. The supercooling $(t\equiv t_*)$ and superheating $(t\equiv t_{**})$ values of t are equal to $t_*=-1$ and $t_{**}=1/8$, respectively.

The elastic term, describing the elastic penalties for a nonhomogeneous ordering of the system, is modeled by $f_e=\kappa_0|\nabla\phi|^2+\kappa\phi^2|\nabla\vec{m}|^2$, where ∇ stands for the gradient operator. The positive elastic constants κ_0 and κ are enforcing homogeneous ordering in ϕ and $\vec{m}$.

The anisotropy random field term is expressed as $f_{\mathrm{ARF}}=-\Lambda P_2(\vec{m}\cdot\vec{e})$, where the positive constant Λ measures the disorder strength tending to align $\vec{m}$ locally along $\vec{e}$. Here the unit vector field $\vec{e}$ randomly varies from site to site and $P_2(\theta)=(3\cos^2\theta-1)/2$ is the Legendre polynomial.

The degree of ordering and the structure of the system for a finite value of Λ depends on the interplay between the introduced free energy contributions. In the following we adopt the Imry-Ma argument [3] which claims that for any strength of the disorder the system breaks into a domain-type pattern, characterized by a dominating average domain size ξ_d. We assume that within each domain, characterized by an average orientation $\vec{m}_d$, the orientational ordering gradually changes on crossing a domain. We thus set $|\nabla\vec{m}|^2\sim 1/\xi_d^2$, reflecting a typical response of a hydrodynamic-type field over a geometrically available space. We further assume that on average the distortions at the domain boundaries are not too high (i.e., the drop of ordering there can be neglected). Consequently we discard in f_e the contribution arising from $|\nabla\phi|^2$.

With this in mind we can estimate the structure and degree of ordering of the system by focusing only on an average free energy density of an average domain of characteristic linear size ξ_d: $f^{(i)}\sim f_b^{(i)}+\kappa\phi^2/\xi_d^2-\Lambda\phi\overline{P_2}$. Note that $\overline{P_2}$ averages to zero for an isotropic distribution of $\vec{e}$ within a domain, which is in practice realized for $\xi_d\to\infty$. For a finite value of ξ_d the central limit theorem suggests $\overline{P_2}\sim 1/\sqrt{N}$, where $N\sim(\xi_d/\xi_r)^3$ counts the number of random sites within a cluster. Here ξ_r estimates a distance on which a random variation in $\vec{e}$ takes place. We set $\xi_r=1$ and henceforth all the distances are measured in units of ξ_r. In order to avoid singularity in $f^{(i)}$ for $\xi_d\to 0$ we introduce the length $\xi^2=\xi_d^2-\xi_r^2=\xi_d^2-1$ which we henceforth treat as a variational variable. Therefore $\xi=0$ signals that ξ_d equals ξ_r. Note that this step does not introduce any additional qualitative features into the problem and introduces only minor quantitative changes.

With this in mind the effective free energy density of the system simplifies to

$$f^{(i)} = f_b^{(i)} + \frac{\kappa\phi^2}{1+\xi^2} - \frac{\Lambda\phi}{(1+\xi^2)^{3/4}}. \tag{3}$$

We continue by studying the phase ordering, represented by variational parameters ϕ and ξ.

The $f_b^{(i)}$ contribution reveals the competition between ordering and *thermal* disordering fluctuations. The f_{ARF} term is a source of additional *static* disorder. This term introduces a kind of frustration into the system. Its local tendency is to enhance the ordering because $f_{\mathrm{ARF}}\propto-\phi$. However it also enforces $\xi\mapsto 0$, which introduces elastic penalties into the system, that tend to decrease the degree of ordering.

Minimizing Eq. (3) with respect to ξ leads to

$$\xi = \begin{cases} 0 & \text{when } \phi<\phi_c, \\ [(\phi/\phi_c)^4-1]^{1/2} & \text{when } \phi>\phi_c, \end{cases} \tag{4}$$

where $\phi_c=3\Lambda/(4\kappa)$ stands for the crossover value of the order parameter.

The resulting free energy can be now expressed as a function of ϕ only:

$$f^{(i)} = \begin{cases} f_p^{(i)} = f_b^{(i)} - \Lambda\phi + \kappa\phi^2 & \text{when } \phi<\phi_c, \\ f_s^{(i)} = f_b^{(i)} - (27/256)(\Lambda/\kappa)^4\phi^{-2} & \text{when } \phi>\phi_c. \end{cases} \tag{5}$$

The subscripts p and s stand for the *para-ordered* and *spero-ordered* phase, respectively, referring to the phase ordering

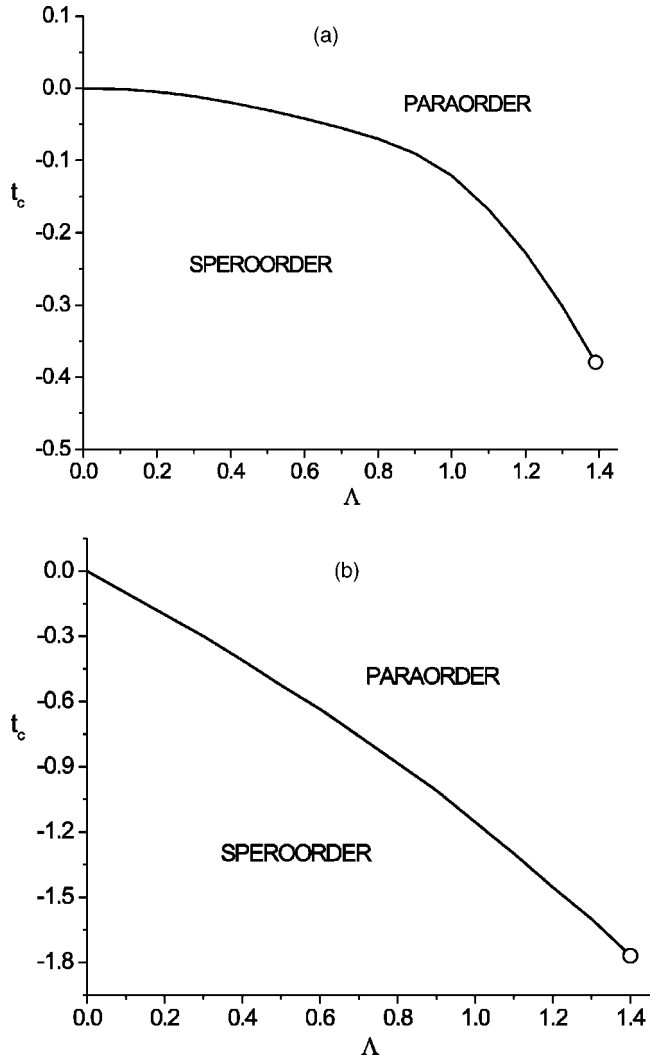

FIG. 1. The critical value t_c of the transition driving control parameter as a function of random field disorder strength Λ. The cases (a) and (b) correspond to condensation potentials $f_b^{(1)}$ [Eq. (1)] and $f_b^{(2)}$ [Eq. (2)], exhibiting (a) discontinuous and (b) continuous phase transitions in unperturbed samples; $\kappa=1$.

for $\Lambda>0$. The para-ordered phase refers to a weakly ordered isotropic phase with $\xi=0$. The spero-ordered phase represents an ordered phase with a finite value of ξ. The terminology we use is analogous to the one used for liquid crystals. There the para- and spero-ordered phases stand for the paranematic and speronematic phases [9], respectively.

We study numerically the phases transition between para-ordered and spero-ordered phases, taking place at $t_c=t_c(\Lambda,\kappa)$. The resulting phase diagrams in the (Λ,t) plane, calculated for $\kappa=1$, are shown in Fig. 1. In Fig. 2 we plot the jump $\Delta\phi$ of the order parameter at transition lines, measuring the strength of the first order phase transitions. The average size of domains is shown in Fig. 3. We note that the first order case has already been treated in detail for the *I-N* transition in liquid crystals [9,11].

In both cases with increased strength Λ the critical vale $t_c(\Lambda)$ monotonically decreases if $\Lambda<\Lambda_c^{(i)}$, where $\Lambda_c^{(1)}\sim 1.39$ and $\Lambda_c^{(2)}\sim 1.40$. Above the critical strength $\Lambda_c^{(i)}$ the transi-

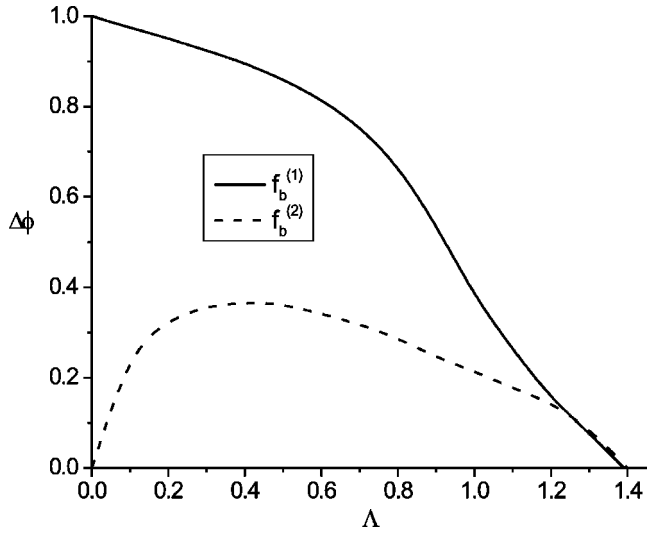

FIG. 2. The strength of discontinuous transitions as a function of Λ. The first (full line) and second (dashed line) order cases are superimposed for comparison; $\kappa=1$.

tions become gradual. Therefore for $\Lambda>\Lambda_c^{(i)}$ a gradual evolution between the para-ordered and spero-ordered phases takes place. A remarkable result of this study is that the disorder converts the second order transition into a discontinuous one, provided that $0<\Lambda<\Lambda_c^{(2)}\sim 1.40$. The first order transition character for $\Lambda>0$ stems from the competition between the condensation ($f_b^{(2)}$) term, which is ready to establish an ordered state for $t<0$, and the disordering term (f_{ARF}). In the $0<\Lambda<\Lambda_c^{(2)}$ regime these terms are comparable. Within the interval $(0,\Lambda_c^{(2)})$ there are two distinct regimes. For a weak enough disorder ($\Lambda<0.4$) the strength of the transition is an increasing, and above this regime a decreasing function of Λ. For $\Lambda>\Lambda_c^{(2)}$ the noncritical disorder contribution is strong enough to wash out any discontinuity as t is varied.

To conclude we compared the influence of the random anisotropy field on the first and second order phase transi-

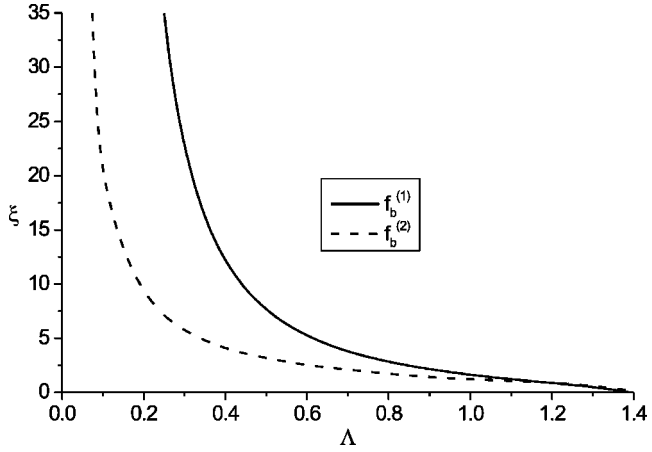

FIG. 3. The domain length of the ordered phase at the transition line. Full and dashed line mark the discontinuous and continuous cases, respectively; $\kappa=1$.

tions. In both cases a strong enough disorder converts the transitions into a gradual evolution of order as t is lowered. Note that there have been numerous studies showing influences of different agents causing a first↔second order phase transition conversion. Halperin, Lubensky, and Ma [14] have shown that a coupling between an order parameter and fluctuations in a hydrodynamic-type field can trigger a second→first conversion. In this case hydrodynamic-type field fluctuations effectively introduce a ϕ^3 term in an $f_b^{(2)}$-type condensation potential. In addition a second→first perturbation induced crossover can be expected in systems close to the tricritical point. In this case in the nonperturbed system is characterized by a relatively weak positive ϕ^4-term contribution, which necessitates also the presence of a positive ϕ^6 term in $f_b^{(2)}$. Coupling with impurities [15] or stress [16] can effectively introduce a contribution proportional to $-\phi^4$. The relative competition between ϕ^4-type contributions now decides the character of the transition. Our study suggests an additional source of the second→first phase transition conversion. With this respect the study carried out by Aharony [17] on randomly perturbed magnets is worth mentioning. He was using the Harris, Plischke, and Zuckermann Hamiltonian [18] and the renormalization-group technique. He observed that uniaxial random perturbations can destabilize the fixed point describing the continuous phase transition in the unperturbed system. He claimed that this instability is the signal of a smeared transition. However, this can be caused also by a first order transition [19].

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